The solution to the problem of the QoS characteristics definition for self-similar traffic serviced by the W/M/1 QS

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***Abstract* – The problem of QoS characteristics definition for self-similar traffic for queuing system of the type W/M/1 using the Weibull distribution is considered. To solve this problem, a transformation of Laplace-Stieltjes is applied. The values of service quality characteristics for self-similar traffic QS W/M/1 such as: the average time of packets delay, the average number of requirements in QS and the length of the packet queue are obtained.**

Keywords – self-similar traffic, Hurst parameter, Weibull distribution, queuing system, the transformation of the Laplace-Stieltjes.

I. INTRODUCTION

The development of modern telecommunications is connected with the transition to the next generation networks *(NGN)*, that have properties of multiservice, multiprotocol and invariance to the switching technology. By its nature, the traffic, serviced in NGN network, is heterogeneous, as it is formed by many but different in their characteristics sources of services and network applications, ensuring the provision of services for voice, data and video *TPS (Triple Play Service).*

The packet traffic of NGN network has specific structure, defined by train character and considerable quantity of pulses. It often causes the admissible network nodes overloading. Hence, particular attention in servicing packet traffic is paid to the quality of service *(QoS)* characteristics support [1, 2].

Packet traffic of NGN network has self-similar (fractal) nature [3-5], the main cause of which is the existence of a long-term dependence between the moments of package arrival, defined by the correlation function in different points in time. Then, as a quantitative evaluation of the degree of self-similarity Hurst parameter H, $0{,}5 \le H < 1$, is used. To describe the self-similar traffic, considering that the moments of packets arrival have the distribution with "heavy-tailed", often use the distribution of Pareto, lognormal or Weibull [3].

At the stage of network hardware and software designing, when it is defined the choice of network structure and the performance of the network nodes and the further operation of NGN QoS, to determine the quality characteristics, it is necessary to apply calculation methods that take into account the self-similarity of packet traffic.

The significant number of different authors is devoted to the QoS characteristic evaluating for the self-similar traffic [3-5]. Most of the works are based on experimental data or results of simulation, however, the reliable and recognized analytical solutions of QoS characteristic evaluating of self-similar traffic for different distributions have been not received. And only for some cases of describing the traffic with different distributions, for example, the gamma distribution, approximate solutions for certain types of queuing systems (QS) have been obtained [8].

In the works to evaluate the QoS characteristics of self-similar traffic the formula Pollaczek-Khinchin for QS of the M/G/1 type, the formula of Norros for QS of the fBM/D/1/∞ type were used [6-8].

All of the above leads to the conclusion about the interest to the matter of finding the QoS characteristics of the self-similar traffic, described by the various types of distributions. This is necessary at the stage of selecting hardware and software means of NGN network, and the structure and size of the buffer devices of network nodes.

The aim of this work is to obtain expressions for finding QoS characteristics for self-similar traffic, described by using the Weibull distribution for QS of the G/M/1 type.

II. CALCULATION OF QoS CHARACTERISTICS OF SELF-SIMILAR TRAFFIC

Let we consider a queuing system (QS) of the G/M/1 type, which serves the flow of requests, where the intervals between requests are described by the random distribution of G, service time has an exponential distribution F, QS is a single-line one [3].

Taking into account that given QS is characterized by a random distribution G of the requests arrival for servicing, the task of calculating the QoS characteristics is significantly complicated. This is due to the fact that such QS requires setting specific distribution law and it is used more often the Pareto, the Weibull or lognormal distribution [3]. Until now, this problem was considered for self-similar traffic with a gamma distribution and solved analytically [8]. The approximate solutions with a certain error were obtained by using simulation for self-similar traffic with distribution of Weibull and Pareto [4-7].

Let we consider the Weibull distribution, which is most unique characteristic for traffic of high-speed data and voice. Let we apply a different approach, namely, we find the Laplace-Stieltjes transformation for the Weibull distribution in order to obtain QoS characteristics of self-similar traffic.

Let we consider the Weibull distribution, given by the differential distribution function [3]:

$$f(x)=\begin{cases}\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}, & x\geq 0,\\ 0, & x\leq 0\end{cases} \qquad (1)$$

where α – parameter of curve form distribution, 0<α<1; α = 2 – 2H, H – Hurst parameter, 0,5≤H<1, $\beta=\left[\lambda\Gamma\left(1+\frac{1}{\alpha}\right)\right]^{-1}$ – distribution parameter, β>0, λ – intensity of requests arrival for service in QS, $\Gamma(k)$ – gamma-function of Euler of the type $\Gamma(k)=\int_{0}^{+\infty}t^{k-1}e^{-t}dt$.

The main characteristics of Weibull distribution are [3]:

– mathematical expectation

$$M=\beta^{-\frac{1}{\alpha}}\Gamma\left(1+\frac{1}{\alpha}\right) \qquad (2)$$

– dispersion

$$D=\beta^{-\frac{2}{\alpha}}\left[\left(\Gamma\left(1+\frac{2}{\alpha}\right)-\Gamma^{2}\left(1+\frac{1}{\alpha}\right)\right)\right] \qquad (3)$$

From [9-10] it is known that for QS of the W/M/1 type the probability of that newly arrived request will find n requests in service within QS is defined as:

$$r_n=(1-\sigma)\sigma^{n},\ 0\leq\sigma<1, \qquad (4)$$

where σ – equation root

$$\sigma=F(\mu-\mu\sigma),\ 0\leq\sigma<1. \qquad (5)$$

Furthermore F – transformation of the Laplace-Stieltjes (LST) of intervals distribution function between requests in QS and has the form

$$F(s)=\int_{0}^{+\infty}e^{-st}f(t)dt,$$

where $f(t)$ – the density of distribution of probability of random value τ (τ – the length of the interval between neighbor requests), μ – intensity of request service in QS.

Having found the roots of σ equation (6), it is possible to define the next quality characteristics for the W/M/1 QS [10]:

– average time of request delay in the system

$$W=\frac{\sigma}{\mu(1-\sigma)}=\frac{\sigma\lambda}{(1-\sigma)}, \qquad (6)$$

where λ – intensity of requests arrival for service in QS,

– the average number of requests in a queue

$$Q=\frac{\rho\sigma}{\mu(1-\sigma)}, \qquad (7)$$

where ρ – coefficient of QS loading,

– the average length of requests queue

$$L=\frac{\rho\sigma}{1-\sigma}. \qquad (8)$$

In the W/M/1 QS the density of distribution of length τ of intervals of requests arrival to QS has the form:

$$f(t)=\alpha\beta t^{\alpha-1}e^{-\beta t^{\alpha}},\ t\geq 0,\ 0<\alpha<1, \qquad (9)$$

where $\beta=\left[\lambda\Gamma\left(1+\frac{1}{\alpha}\right)\right]^{-1}$ – distribution parameter, β>0, Γ – gamma-function, λ – intensity of requests arrival for service in QS.

Let we write the expression (10) in the form:

$$\sigma=\alpha\beta\int_{0}^{+\infty}e^{-(\mu-\mu\sigma)t}t^{\alpha-1}e^{-\beta t^{\alpha}}dt. \qquad (10)$$

Using the expansion of a function e^x in a Maclauren's series [11] $e^{x}\approx\sum_{n=0}^{\infty}\frac{x^{n}}{n!}$, where $x\in(-\infty;+\infty)$, the expression (9) takes the form:

$$f(t)\approx\alpha\beta t^{\alpha-1}\sum_{n=0}^{\infty}\frac{(-\beta t^{\alpha})^{n}}{n!},$$

or

$$f(t)\approx\sum_{n=0}^{\infty}\frac{\alpha(-1)^{n}\beta^{n+1}t^{(n+1)\alpha-1}}{n!},$$

hence

$$f(t)\approx\sum_{n=1}^{\infty}\alpha\frac{(-1)^{n-1}}{(n-1)!}\beta^{n}t^{n\alpha-1}. \qquad (11)$$

For the original $f(t)$, defined by (11), it is necessary to find the image F. It is known that the Laplace transformation has linear property for a finite number of terms [12-13]. If $f_1(t)$, $f_2(t)$, ..., $f_N(t)$ – pro image, hence $\forall c_i\in C$, i = 1,N, function $f(t)\approx\sum_{k=1}^{N}c_k f_k(t)$ is also the pro image and the equation is correct:

$$L\left[\sum_{k=1}^{N}c_k f_k(t)\right]=\sum_{k=1}^{N}c_k L[f_k(t)], \qquad (12)$$

where L – operator of Laplace.

Therefore, we find the image for the original $f(t)$:

$$f(t)\approx\alpha\sum_{n=1}^{N}\frac{(-1)^{n-1}}{(n-1)!}\beta^{n}t^{n\alpha-1}. \qquad (13)$$

In the transition under the set conditions *f(t)* from infinite sum to finite value *N* it is necessary to consider the error value that is:

$$R_n = \alpha \left| \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} \beta^n t^{n\alpha-1} - \sum_{n=1}^{N} \frac{(-1)^{n-1}}{(n-1)!} \beta^n t^{n\alpha-1} \right|. \quad (14)$$

From the table of Laplace transformation [11-12] for the pro image $f(t) = t^{\alpha}$, $(\alpha > -1)$ let we find the image

$$F(p) = \frac{\Gamma(\alpha+1)}{p^{\alpha+1}}, \quad (15)$$

where Γ – gamma-function.

Therefore, applying the property (12) for the pro image (11), we will obtain the image *F(p)* and it takes the form:

$$F(p) \approx \sum_{n=1}^{N} \alpha \frac{(-1)^{n-1}}{(n-1)!} \beta^n \frac{\Gamma(n\alpha)}{p^{n\alpha}}, \quad n\alpha > 0 \quad (16)$$

where Γ – gamma-function.

Then the expression (16), considering the fact that $p=\mu-\mu\sigma$, has the form:

$$F(\mu-\mu\sigma) \approx \sum_{n=1}^{N} \alpha \frac{(-1)^{n-1}}{(n-1)!} \beta^n \frac{\Gamma(n\alpha)}{(\mu-\mu\sigma)^{n\alpha}}, \quad n\alpha > 0 \quad (17)$$

where Γ – gamma-function.

The equation (5) takes the form:

$$\sigma \approx \sum_{n=1}^{N} \alpha \frac{(-1)^{n-1}}{(n-1)!} \beta^n \frac{\Gamma(n\alpha)}{(\mu-\mu\sigma)^{n\alpha}}, n\alpha > 0 \quad (18)$$

where Γ – gamma-function.

Solving the equation (18) we find the root of equation σ. It allows defining the necessary quality characteristics for the G/M/1 QS [14-16]:

– the value of average time delay W of the request in the QS (formula 7),

– the average number of requests Q in the queue of QS (formula 8),

– the average queue length of requests L (formula 9).

CONCLUSIONS

1. The solution of defining the QoS characteristics of self-similar traffic for the queuing system of the W/M/1 type by using the Weibull distribution is considered.

2. The values of quality of service characteristics for self-similar traffic for the W/M/1 QS, such as the average packet delay time, the average number of requirements in QS and the packet queue length are obtained.

3. The obtained results allow on the stage of designing and further operation of hardware software means of NGN networks to consider the specifics of building and structure of network nodes in the conditions of real processes of the operation.

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